Pole Shape Optimization for Radiation Resistant Quadrupole Magnets of the SIS-100 Accelerator

A. Kalimov¹, H. Leibrock², C. Muehle², P. Nalimov¹

¹St. Petersburg State Polytechnic University, 195251, St. Petersburg, Russia ²GSI Helmholtzzentrum für Schwerionenforschung GmbH Darmstadt, D-64291, Germany

The SIS-100 is a charge particle accelerator developed as a part of the challenging international project Facility for Anti-proton and Ion Research (FAIR) taking place in Darmstadt, Germany. The ion optics of the SIS-100 accelerator includes superconducting and normal conducting quadrupole magnets. To fulfill the requirements of the field quality in the magnet aperture it is necessary to find an optimal pole tip shape as well as an optimal configuration of the coil system. We used a specially developed optimization procedure for designing the magnet cross section. The pole border line is described by a superposition of hyperbolic functions corresponding to different Fourier components of the magnetic field expansion. Strong correlation between amplitudes of the pole shape and field harmonics enables high performance of the optimization algorithms. The developed procedures have been used for designing a quadrupole magnet with especially wide range of the flux density variation.

*Index Terms***— Quadrupole magnets, accelerator magnets, magnetic field, pole shape optimization, field quality.**

I. INTRODUCTION

ULTIPOLE magnets are widely used in qaccelerators and MULTIPOLE magnets are widely used in qaccelerators and lines for transporting beams of charged particles. The magnets with the quadrupole rotational symmetry of the magnetic field are used in focusing systems in such facilities [1]. Requirements to the field quality in the aperture of the magnets depend strongly on their real application. In charged particle accelerators the deviation of the magnetic flux density inside the "good field area" from the ideal dependence typically should not exceed several units of 10^{-4} . To provide the acceptable field distribution a magnet designer should find appropriate positions of the coils and the optimal shape of the iron poles with a very high precision.

A new ion accelerator SIS-100 is developed in frames of challenging project FAIR in Darmstadt, Germany [2]. This accelerator is equipped with superferric magnets consisting of iron yokes and superconducting coils [3]. The exclusion is made for several ion optical elements placed at the entrance of the accelerator where the high radiation does not allow to use superconductors. Special quadrupole magnets were developed to be installed in this area. Due to critical shortage of the free space in the accelerator ring the flux density in the aperture of these elements was increased strongly compared to conventional quadrupole magnets. Usually the pole tip flux density in the quadrupole magnets does not exceed a threshold of $0.9 - 1.0$ T because saturation effects in the iron poles distorts the field distribution in the aperture noticeably. It is important to note that the high field quality in the accelerator magnets is required for the wide range of the excitation current variation. Main requirements formulated for the radiation resistant quadrupole magnets on the basis of ion optical calculations (see Table 1) include enormously high flux density in the aperture. A design of this magnet required multi-object optimization of its cross section including a proper choice of the pole shape.

TABLE I MAIN REQUIREMENTS TO THE SIS-100 RADIATION RESISTANT QUADRUPOLE MAGNETS

Parameter	Unit	Value
Pole tip radius	mm	66
Magnet length	m	1.76
Maximum pole tip flux density	т	1.35
Maximum field gradient	T/m	20
Good field area radius	mm	55
Field quality	$\frac{0}{0}$	± 0.1

II. POLE SHAPE OPTIMIZATION

Strategy of the pole shape optimization in the quadrupole magnet is based on a presentation of the field characteristics as a superposition of the Fourier harmonics. The vector magnetic potential $U(r, \theta)$ may be expressed in polar coordinates with *g* being the pole tip radius by an expansion:

$$
U(r,\theta) = U_2 \left(\frac{r}{g}\right)^2 \cos(2\theta) + U_6 \left(\frac{r}{g}\right)^6 \cos(6\theta) \dots (1)
$$

For a system with the quadrupole fourfold symmetry only the coefficients U_2 , U_6 , U_{10} , ... should be included in the expansion. Moreover, for the ideal magnet only the first term in (1) with the amplitude of U_2 should not be equal to zero. In practice the ideal field configuration is disturbed by the finite dimensions of the pole, by the influence of coils, nonlinear magnetic properties of the steel etc. which all excite highorder harmonics. Amplitudes of these harmonics may be decreased strongly by proper choice of the pole tip shape. We developed a fast and reliable optimization procedure to solve this problem. The main idea of this procedure is a compensation of the high order field harmonics by appropriate deformation of the pole profile [4] - [6]. The pole border line may be also described in terms of Fourier series. Terms of this series exactly correspond to the field expansion for the

infinitely wide pole with infinitely big magnetic permeability and may be defined by solving the equation (2), which considers the pole border as a line of the constant magnetic potential:

$$
U(r,\theta) = \sum_{j=1}^{J} P_j \cdot (r/g)^j \cdot \cos(j \cdot \theta) = const \qquad (2)
$$

Each of the parameters P_i defines the amplitude of one pole shape harmonics. In the real situation amplitudes of the pole shape harmonics and field harmonics differs, nevertheless strong correlation between them takes place. This property of the considered problem is used to build fast and robust optimization algorithm for the pole shape definition. The flux density in the aperture of the magnet is defined by appliing a finite element based procedure for the vector magnetic potential. For two-dimensional problems the vector magnetic potential has only one component and in the current free aperture of the magnet may be easily transformed to the scalar magnetic potential. This transformation is used to find pole shape harmonics defined by (2). The described optimization procedure for the quadrupole magnets is integrated in the software *MULTIMAG* developed for computing magnetic fields in accelerator magnets [7].

III. 2D DESIGN OF THE MAGNET

The developed optimization procedure is applied to find optimal shape of the pole tip of the quadrupole magnet. The main difficulty of the considered problem is achieving acceptable field quality for both low and high flux density levels. For this purpose the goal function for the pole shape optimization at low excitation current was composed of positive non-zero amplitude of the 6-th harmonic and zero amplitudes of the higher ones. Saturation effects decrease first of all the amplitude of the 1-st undesired harmonic (6-th in our case). So we can keep the field quality in requested margins in relatively wide range of the magnetic flux density variation.

Fig.2.Flux density distribution at the border of the good field area for the low and high flux density in the aperture.

In the high fields the amplitude of the 6-th harmonic changes sign but stays acceptable (see Fig. 2 and Table II).

TABLE II MAIN FIELD HARMONICS IN THE MAGNET APERTURE

	Amplitude	Amplitude
Harmonic number	$G = 2$ T/m	$G = 20$ T/m
	10^{-4} rel. units	10^{-4} rel. units
	8.02	-9.06
10	0.003	-2.58
14	0.001	-0.003
18	-0.17	-0.14
22	-0.001	0.002

Comparison between the hyperbolic pole shape and the optimized one is shown in Fig. 3.

Fig.2. Cross section of the pole tip before (*a*) and after (*b*) optimization

3D optimization of the considered magnet was undertaken as well. It was demonstrated that the main integral properties of the magnetic field in the aperture are similar to characteristics obtained by the magnet cross section.

IV. MAIN RESULTS

The optimization procedure for the pole shape definition is developed and applied for the designing of the SIS-100 accelerator radiation resistant magnets. A very wide range of the flux density variation was provided by artificial deterioration of the field quality in low magnetic fields. The maximum pole tip flux density of 1.35 T with acceptable field quality in the aperture is obtained for the maximum excitation current.

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